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A new equation for quantum dynamics: Explanation for wave-particle duality, relativistic derivation from quantum mechanics and law of relativistic property

Research Article

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Abstract: Using the Schrodinger equation we were able to predict the nature of the quantum world in the most accurate way. This paper begins with deriving a new equation, which can relate to wave-function, velocity position, and mass of the quantum system. The derived equation can have many uses in theoretical and experimental approaches of physics. One use is already discussed in this paper. That use is, predicting the nature of the quantum system by using a usual plane wave-function. This prediction of the nature of the quantum system can help us understand the reason for the duality of the quantum system, behaving as both wave and particle. The next part of the paper is a bit abstract one but the most promising breakthrough of this paper. By using the information given by our derived equation, we can derive the famous Einstein's relativistic equations for mass, length and time, on a condition of constant velocity or no force. This way, this paper also connects quantum mechanics with the special theory of relativity. The final work of the paper will be giving a definition or a law, to state which kind of property can be relativistic and which cannot be a relativistic property.

Keywords: quantum dynamics • relativistic quantum mechanics • wave-particle duality • applications of Schrodinger equation • special theory of relativity

1. Introduction

One of the important aspects of quantum mechanics is using Operators. There have been a few attempts on introducing force operator [1] and this paper does not defy them, but it introduces a better version of force operator, which leads us to some useful results. Relative to other operators, force operator has been the least discussed force and it is discussed on very rare concepts like the quantum Langevin equation by G.W.Ford and M.Kac [2]. This paper begins with defining a new form of an operator for Force. Using the new derived force operator, we can find a new equation for quantum mechanics. We will use the Schrodinger equation for deriving

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our final equation. We will use the plane wave-function equation to derive a supporting result in order to prove the derived equations. Among the results, we will use one of the derivations to explain wave-particle nature. There can be much further uses of those equations and one very interesting use is given in the paper which intends to connect relativity in order to derive a law which can help us to find the relativistic or non-relativistic nature of a property. There are has been a huge amount of efforts for the collaboration of relativity and quantum mechanics [3], though not a satisfying result has been found yet, this paper is one of those efforts. The derivations related to relativity will use some abstract methods to derive proper results, which could lead us to the law for the relativistic property. The relativity related part of this paper is also an effort to connect relativity to quantum mechanics, which has been one of the most discussed topics in physics since last few decades. Also, note that the manuscript is studied in one dimension.

2. Methods

We know that,

$$F = \frac{dp}{dt} \tag{1}$$

So, it can be defined in the form ,

$$F = \frac{\hat{E}p}{i\bar{h}} \tag{2}$$

as,

$$\hat{E} = i\bar{h}\frac{d}{dt} \tag{3}$$

So, we can define a force operator,

$$\hat{F} = \frac{\hat{E}\hat{p}}{i\bar{h}} \tag{4}$$

Then,

$$\hat{F}\varphi = \frac{\hat{E}\hat{p}}{i\bar{h}}\varphi \tag{5}$$

Suppose, E as total energy, U as potential energy, K as kinetic energy and W as work done by external force.

$$E = K + U = W + U \tag{6}$$

So,

$$Fx = -(U - E) \tag{7}$$

We may define Fx as an operator too,

$$-\hat{F}\hat{x} = \hat{U} - \hat{E} \tag{8}$$

We will use the previously derived force operator,

$$\hat{F}\hat{x}\varphi = -\frac{\hat{E}\hat{P}\hat{x}}{i\bar{h}}\varphi \tag{9}$$

$$(\hat{U}-\hat{E})\varphi = -\frac{\hat{E}\hat{P}\hat{x}}{i\bar{h}}\varphi \tag{10}$$

$$i\bar{h}(\hat{U}-\hat{E})\varphi = -\hat{E}\hat{P}\hat{x}\varphi \tag{11}$$

$$i\bar{h}\hat{U}\varphi - i\bar{h}\hat{E}\varphi = -\hat{E}\hat{P}\hat{x}\varphi \tag{12}$$

$$i\bar{h}\hat{U}\varphi - i\bar{h}(i\bar{h}\frac{d\varphi}{dt}) = -\hat{E}\hat{P}\hat{x}\varphi \tag{13}$$

$$i\bar{h}\hat{U}\varphi - i\bar{h}(i\bar{h}\frac{d\varphi}{dt}) = -h^2\frac{d}{dt}\frac{d}{dx}x\varphi \tag{14}$$

$$\hat{U}\varphi - i\bar{h}\frac{d\varphi}{dt} = i\bar{h}\frac{d}{dt}(\varphi + x\frac{d\varphi}{dx}) \tag{15}$$

$$\hat{U}\varphi - i\bar{h}\frac{d\varphi}{dt} = i\bar{h}\frac{d\varphi}{dt} + i\bar{h}\frac{dx}{dt}\frac{d\varphi}{dx} + i\bar{h}x\frac{d}{dt}\frac{d\varphi}{dx} \tag{16}$$

$$\hat{U}\varphi = 2i\bar{h}\frac{d\varphi}{dt} + i\bar{h}\frac{dx}{dt}\frac{d\varphi}{dx} + i\bar{h}x\frac{d}{dt}\frac{d\varphi}{dx} \tag{17}$$

We found an equation for the operator of potential energy. Using schcrodinger equation,

$$i\bar{h}\frac{d\varphi}{dt} = -\frac{\bar{h}^2}{2m}\frac{d^2\varphi}{dx^2} + \hat{U}\varphi \tag{18}$$

Inserting the equation of potential energy operator in schrodinger equation,

$$i\bar{h}\frac{d\varphi}{dt} = -\frac{\bar{h}^2}{2m}\frac{d^2\varphi}{dx^2} + 2i\bar{h}\frac{d\varphi}{dt} + i\bar{h}\frac{dx}{dt}\frac{d\varphi}{dx} + i\bar{h}x\frac{d}{dt}\frac{d\varphi}{dx} \tag{19}$$

We find this equation,

$$0 = -\frac{h^2}{2m}\frac{d^2\varphi}{dx^2} + i\bar{h}\frac{d\varphi}{dt} + i\bar{h}\frac{dx}{dt}\frac{d\varphi}{dx} + i\bar{h}x\frac{d}{dt}\frac{d\varphi}{dx} \tag{20}$$

By simplifying this equation,

$$0 = \frac{i\bar{h}}{2m}\frac{d^2\varphi}{dx^2} + \frac{d\varphi}{dt} + v\frac{d\varphi}{dx} + x\frac{d}{dt}\frac{d\varphi}{dx} \tag{21}$$

We found an equation which can give us new information about the quantum system. So we have found a new equation, which can be said to be a modified form of Schrodinger equation, which relates between displacement, velocity and the wave function.

2.1. Proof for the equation

Suppose wave function as a plane wave, defined as follows.

$$\varphi = e^{i(kx-\omega t)} = e^{\phi} \tag{22}$$

Then,

$$\frac{d^2\varphi}{dx^2} = -k^2\varphi, \frac{d\varphi}{dx} = ik\varphi, \frac{d\varphi}{dt} = i\omega\varphi \tag{23}$$

Now we will use these terms into our found equation,

$$0 = \frac{i\bar{h}}{2m}\frac{d^2\varphi}{dx^2} + \frac{d\varphi}{dt} + v\frac{d\varphi}{dx} + x\frac{d}{dt}\frac{d\varphi}{dx} \tag{24}$$

$$0 = \frac{i\bar{h}}{2m}(-k^2\varphi) + (-i\omega\varphi) + v(ik\varphi) + xk\omega\varphi \tag{25}$$

$$0 = -\frac{i\bar{h}k^2}{2m} - i\omega + ivk + xk\omega \tag{26}$$

We will equate both imaginary and real part equal to Zero.

Imaginary part;

$$0 = -\frac{\bar{h}k^2}{2m} - \omega + vk \tag{27}$$

Real Part;

$$xk\omega = 0 \tag{28}$$

Now for the imaginary part,

$$\frac{\bar{h}k^2}{2m} + \omega = vk \tag{29}$$

$$\frac{\bar{h}k}{2m} + \frac{\omega}{k} = v \tag{30}$$

$$\frac{\bar{h}k}{2m} = v - \frac{\omega}{k} \tag{31}$$

$$\frac{\bar{h}k}{2m} = v_g - v_p \tag{32}$$

The difference between Phase velocity and the group or classical velocity is exactly the same as the value currently accepted by quantum mechanics of our previous knowledge. *Hence, it is proved that the equation is correct, but still, the necessity of experimental proof cannot be ignored.*

For the real part,

$$xk\omega = 0 \tag{33}$$

$x \neq 0$, which means, $k\omega = 0$. This can be understood by following cases,

Case 1; $k = 0$ and $\omega = 0$

If $k = 0$, then in the following equation,

$$v_p = \frac{\omega}{k} \tag{34}$$

Phase velocity will not be defined which is not possible in a quantum system. So, this case is false.

Case 2; $K = 0$

Because of the Same reason as case1, this case is also false.

Case 3; $\omega = 0$

In this case

$$\frac{\bar{h}k}{2m} = v_g \tag{35}$$

and

$$v_p = 0, \tag{36}$$

this does not oppose any property of the quantum system, so this case must be the true one.

2.2. The explanation for Wave-particle duality

When x is negligible, there is no issue with phase velocity, but when x is considerable phase velocity becomes zero according to our equation.

$$xk\omega = 0 \tag{37}$$

This can be a key to understand the wave-particle nature of quantum states. To understand this, let us revise youngs double slit experiment, which also works for electrons and before few years such experiment was also done to buckyballs, which showed similar results of bright and dark fringes. According to our derived equation, the state whether it is an electron or a buckyball behaves like a proper wave at the moment when a field or force is applied. But in the next moment, the frequency and the phase velocity reaches zero. Due to this reason, the wave nature vanishes, so the state must behave like a particle. In Young's experiment, when the particle reaches the slit, the force field is created due to the surrounding material of the slit. According to our equations, It is not dependent on the strength of the force, but just when the force acts, it properly behaves as a wave. So the motion of the state transforms just as the motion of a wave would be in the slit. But at the next moment, the state behaves as a particle, so when the state is detected at the screen and it shows the particle nature. This explanation can be a key to understand the good old wave-particle duality.

2.3. Derivation of Relativistic equation from quantum mechanics

If ω=0, then

$$\frac{d\varphi}{dt} = -i\omega\varphi = 0 \tag{38}$$

and

$$\frac{d}{dt}\frac{d\varphi}{dx} = k\omega\varphi \tag{39}$$

Then our derived equation becomes,

$$0 = \frac{i\bar{h}}{2m}\frac{d^2\varphi}{dx^2} + v\frac{d\varphi}{dx} \tag{40}$$

and

$$v = \frac{\bar{h}k}{2m} \tag{41}$$

Then,

$$\frac{i\bar{h}}{2m}\frac{d^2\varphi}{dx^2} + (\frac{\bar{h}k}{2m})\frac{d\varphi}{dx} = 0 \tag{42}$$

On simplification,

$$i\frac{d^2\varphi}{dx^2} + k\frac{d\varphi}{dx} = 0 \tag{43}$$

Integrating the equation with respect to dx,

$$\int (i\frac{d^2\varphi}{dx^2} + k\frac{d\varphi}{dx})dx = 0 \tag{44}$$

$$i\frac{d\varphi}{dx} + \int (k\frac{d\varphi}{dx})dx = 0 \tag{45}$$

$$i\frac{d\varphi}{dx} + k\varphi - \int (\frac{dk}{dx}\varphi)dx = 0 \tag{46}$$

$$\frac{d\varphi}{dx} - ik\varphi + i\int \frac{dk}{dx}\varphi dx = 0 \tag{47}$$

$$ik\varphi - ik\varphi + i\int \frac{dk}{dx}\varphi dx = 0 \tag{48}$$

$$\int \frac{dk}{dx}\varphi dx = 0 \tag{49}$$

$$\frac{dk}{dx} = 0 \tag{50}$$

Please, note that further equations are built intentionally to form desired result. This part is an abstract form of work, to realize the possibilities.

2.3.1. Relativistic mass equation

When No Force is acting or when group velocity is constant,

$$v_p = 0, v_g = \frac{\bar{h}k}{2m} \tag{51}$$

So,

$$mv_g = mv = \frac{\bar{h}k}{2m} \tag{52}$$

$$\frac{dv}{dx} = \frac{d}{dx}(\frac{\bar{h}k}{2m}) \tag{53}$$

We know, that

$$\frac{dk}{dx} = 0 \tag{54}$$

Then, we might assume that mass is variant under position,

$$\frac{dv}{dx} = -\frac{\bar{h}k}{2m^2}\frac{dm}{dx} \tag{55}$$

We know from the Newtonian mechanics that,

$$a = v\frac{dv}{dx} \tag{56}$$

Then,

$$a = -\frac{\bar{h}^2k^2}{4m^3}\frac{dm}{dx} \tag{57}$$

$$F = -\frac{\bar{h}^2k^2}{4m^2}\frac{dm}{dx} \tag{58}$$

$$F = -v^2\frac{dm}{dx} \tag{59}$$

When F=0, then

$$\frac{dm}{dx} = 0 \tag{60}$$

Suppose, a term

$$E = mc^2 = (m - m_0)c^2 \tag{61}$$

Here, m(0) is constant, then

$$\frac{dE}{dx} = \frac{dm}{dx}c^2 \tag{62}$$

(note; E is just used as a term and not as any physical interpretation of energy) So,

$$\frac{dE}{dx} = 0 \tag{63}$$

Now,We can build the following terms,

$$(m + m_0)\frac{dE}{dx} + E\frac{dm}{dx} = 0 \tag{64}$$

We already know that,

$$\frac{dk}{dx} = 0 \tag{65}$$

then,

$$\frac{\bar{h}^2k}{2}\frac{dk}{dx} = 0 \tag{66}$$

Combining Equations,

$$(m + m_0)\frac{dE}{dx} + E\frac{dm}{dx} = \frac{\bar{h}^2k}{2}\frac{dk}{dx} \tag{67}$$

$$m_0\frac{dE}{dx}+(m\frac{dE}{dx}+E\frac{dm}{dx})=\frac{\bar{h}^2k}{2}\frac{dk}{dx} \tag{68}$$

$$m_0\frac{dE}{dx}+\frac{d(Em)}{dx}=\frac{\bar{h}^2k}{2}\frac{dk}{dx} \tag{69}$$

$$\frac{d(E(m+m_0))}{dx}=\frac{\bar{h}^2k}{2}\frac{dk}{dx} \tag{70}$$

$$\frac{d(E(m+m_0))}{dx}=\frac{\bar{h}^2}{4}(2k\frac{dk}{dx}) \tag{71}$$

$$\frac{d(E(m+m_0))}{dx}=\frac{\bar{h}^2}{4}\frac{d(k^2)}{dx} \tag{72}$$

$$\frac{d(E(m+m_0))}{dx}=\frac{d(\frac{\bar{h}^2k^2}{4})}{dx} \tag{73}$$

Integrating both sides,

$$E(m+m_0)=\frac{\bar{h}^2k^2}{4} \tag{74}$$

Now,

$$E(m+m_0)=p^2 \tag{75}$$

Putting value of E,

$$(m-m_0)(m+m_0)c^2=m^2v^2 \tag{76}$$

$$(m^2-m_0^2)c^2=m^2v^2 \tag{77}$$

$$m^2-m_0^2=\frac{m^2v^2}{c^2} \tag{78}$$

$$m^2-\frac{m^2v^2}{c^2}=m_0^2 \tag{79}$$

$$m^2(1-\frac{v^2}{c^2})=m_0^2 \tag{80}$$

$$m^2=\frac{m_0^2}{(1-\frac{v^2}{c^2})} \tag{81}$$

$$m=\frac{m_0}{\sqrt{(1-\frac{v^2}{c^2})}} \tag{82}$$

2.3.2. Relativistic length equation

We will suppose a system of two particles having equal velocity and same direction. Note that the external force on the system is supposed to be zero, but internal forces like gravitation and electromagnetism can exist internally. The gravitational force between two particles is negligible. Suppose l as length between two particles.

$$\frac{Gm_1m_2}{l^2} = 0 \tag{83}$$

Diffrentiating both sides,

$$\frac{Gm_1}{l^2}\frac{dm_2}{dx} + \frac{Gm_2}{l^2} - \frac{2Gm_1m_2}{l^2}\frac{dl}{dx} = 0 \tag{84}$$

Also, we have found previously that mass is invariant under position when force is not acting. So,

$$\frac{dm_1}{dx} = \frac{dm_2}{dx} = 0 \tag{85}$$

So, this means

$$\frac{dl}{dx} = 0 \tag{86}$$

Using a similar method of the one, we used for mass, Suppose a term

$$E_l = (l_0 - l)c^2 \tag{87}$$

So diffrentiating it, (l(0) is constant)

$$\frac{dE_1}{dx} = -\frac{dl}{dx}c^2 = 0 \tag{88}$$

Now, we can form the following equations,

$$(l + l_0)\frac{dE_1}{dx} + E_1\frac{dl}{dx} = 0 \tag{89}$$

and

$$2v\frac{dv}{dx}l_0^2 = -\frac{\bar{h}k}{2m^2}\frac{dm}{dx} = 0 \tag{90}$$

Combining both equations,

$$(l + l_0)\frac{dE_1}{dx} + E_1\frac{dl}{dx} = 2v\frac{dv}{dx}l_0^2 \tag{91}$$

Simplifying this equation in the similar way we did in the previous derivation,

$$\frac{((l + l_0)E_1)}{dx} = \frac{d(v^2l_0^2)}{dx} \tag{92}$$

$$(l + l_0)E_1 = v^2l_0^2 \tag{93}$$

$$(l + l_0)(l_0 - l)c^2 = v^2l_0^2 \tag{94}$$

$$(l_0^2 - l^2)c^2 = v^2l_0^2 \tag{95}$$

$$l = l_0(\sqrt{1 - \frac{v^2}{c^2}}) \tag{96}$$

2.3.3. Relativistic Time equation

To derive the time equation, we will use equations of non-accelerated motion as this case is for constant velocity.This case is the most abstract one but it is still quite derivable.

$$vt = x \tag{97}$$

By diffrentiating on both sides,

$$\frac{dv}{dx}t + v\frac{dt}{dx} = 1 \tag{98}$$

As,

$$a = v\frac{dv}{dx} = 0, so\frac{dv}{dx} = 0 \tag{99}$$

$$v\frac{dt}{dx} = 1, so\frac{dt}{dx} = \frac{1}{v} \tag{100}$$

As we know relativistic effects are only shown when the velocity is in higher amount, so we here we will suppose that velocity is higher enough to consider it's reciprocal equal to zero.

$$\frac{dt}{dx} = \frac{1}{v} \approx 0 \tag{101}$$

But to derive proper equation we have to use t instead of t.

$$\frac{d(\Delta t)}{dx} = 0 \tag{102}$$

Suppose a term

$$E_t = (\Delta t - \Delta t_0)c^2, \frac{dE_t}{dx} = \frac{d\Delta t}{dx}c^2 \tag{103}$$

Using the same method used in previous derivation we will form following equations,

$$(\Delta t + \Delta t_0)\frac{dE_t}{dx} + E_t\frac{d\Delta t}{dx} = 0 \tag{104}$$

and also,

$$2v\Delta t^2\frac{dv}{dx} + 2\Delta t v^2\frac{d\Delta t}{dx} = 0 As, \frac{dv}{dx} = \frac{d\Delta t}{dx} = 0 \tag{105}$$

Now, combining both equations,

$$(\Delta t + \Delta t_0)\frac{dE_t}{dx} + E_t\frac{d\Delta t}{dx} = 2v\Delta t^2\frac{dv}{dx} + 2\Delta t v^2\frac{d\Delta t}{dx} \tag{106}$$

The simplifying right-hand side we get,

$$\frac{d(E_t(\Delta t + \Delta t_0))}{dx} = 2v\Delta t^2\frac{dv}{dx} + 2\Delta t v^2\frac{d\Delta t}{dx} \tag{107}$$

Applying reverse chain rule to the left-hand side,

$$\frac{d(E_t(\Delta t + \Delta t_0))}{dx} = \frac{d(\Delta t^2 v^2)}{dx} \tag{108}$$

Removing differentiation,

$$E_t(\Delta t + \Delta t_0) = \Delta t^2 v^2 \tag{109}$$

$$(\Delta t - \Delta t_0)(\Delta t + \Delta t_0)c^2 = \Delta t^2 v^2 \tag{110}$$

Simplifying the equation,

$$\Delta t = \frac{\Delta t}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{111}$$

The only difference for time equation is that it is applicable for high velocity.

3. Results

We introduced a new form of force operator in terms of energy and momentum operator and used that to derive a new equation. We found an equation for potential energy operator, which can be implemented in practical approaches and experiments.

$$\hat{U}\varphi = 2i\bar{h}\frac{d\varphi}{dt} + i\bar{h}\frac{dx}{dt}\frac{d\varphi}{dx} + i\bar{h}x\frac{d}{dt}\frac{d\varphi}{dx} \tag{112}$$

It is quite possible to find the eigenvalues of potential energy for specific systems. It is to be noted that velocity and position plays a role in determining the eigenvalues of potential energy. Through this paper, I would like to encourage other respective physicist to discover, what more can be known through these equations about various systems. We also found an equation which can give us insight and more information about a quantum state and its dynamics.

This derived equation may act as a support for mathematical models of quantum states, future theoretical discoveries and practical features of quantum technology

$$0 = \frac{i\bar{h}}{2m}\frac{d^2\varphi}{dx^2} + \frac{d\varphi}{dt} + v\frac{d\varphi}{dx} + x\frac{d}{dt}\frac{d\varphi}{dx} \tag{113}$$

We found that the prediction of the difference between phase velocity and group or classical velocity is exactly the same as we have derived in the studies of quantum mechanics. We used the equation and derived more information about the quantum system, which led us to a possible explanation of wave-particle duality. Wave-particle duality has been a good deal of debate among the physics community and it will be cherished to find a proper explanation for the dual behavior. If the equation of this paper gets proved correct, experimentally then the proposed explanation for duality in this paper can act as a strong solution. So, I would also encourage experimental physicists to find a way to test the equation.

4. Discussion

The most amusing part of this paper is the derivation of relativistic formulas of mass, length and time. The relativistic equations getting derived from quantum mechanics itself is quite surprising and unexpected.

Physicist, over the last decade or two, have contributed much time and effort to find relation between quantum theory and relativity. Much success has been achieved, but not a satisfactory one. This paper can act as the next step for the connection between these two opposite banks in the wide river of physics. The method used to derive relativistic results are surely framed in a way to get specific results in form of relativistic equations. We can do this in reverse method, and use relativistic equation, differentiate them with x, and then prove that they are true

We can define in a sense that which properties can be relativistic and which properties cannot be relativistic. Here is how we can define. The necessity for the derivation of all three relativistic equation was its invariance under displacement. We connected the invariance of wave number k and the invariance of the three properties to derive the relativistic equation.

$$\frac{dm}{dx} = 0 ==> m = \frac{m_0}{\sqrt{(1-\frac{v^2}{c^2})}} \tag{114}$$

$$\frac{dl}{dx} = 0 ==> l = l_0(1-\frac{v^2}{c^2}) \tag{115}$$

$$\frac{d\Delta t}{dx} = 0 ==> \Delta t = \frac{\Delta t_0}{\sqrt{(1-\frac{v^2}{c^2})}} \tag{116}$$

So, we can give a definition or law of relativistic property. Only those properties which are invariant with displacement show relativistic nature or in other words for a property H, to be a relativistic property its derivative with respect to displacement x should be equal to zero. Let us take an example, suppose an electrostatic force between two quantum state having equal charge q, behaves as

$$F = \frac{Kq^2}{l^2} \tag{117}$$

So, its derivative,

$$\frac{2Kq}{l^2}\frac{dq}{dx} - \frac{2Kq^2}{l^3}\frac{dl}{dx} = \frac{dF}{dx} \tag{118}$$

where dl/dx=0,from our previous derivations

$$\frac{2kq}{l^2}\frac{dq}{dx} = \frac{dF}{dx} \neq 0 \tag{119}$$

Because electrostatic forces are not negligible for quantum states and also they are variant under position.

So

$$\frac{dq}{dx} \neq 0 \tag{120}$$

Hence, the charge is not relativistic. By this way, we can confirm which property is relativistic and which one is not relativistic. These derivation and explanation of relativistic equation in these equations of quantum mechanics are just from the special theory of relativity. Yet we have no idea about the relationship of the general theory of relativity to this work.

5. Conclusion

After all the derivations and explanation we have gathered the following conclusions,

- We can use the force operator in terms of energy and momentum operator,

$$F = \frac{\hat{E}p}{i\bar{h}} \tag{121}$$

- We found a useful equation for the operator of potential energy,

$$\hat{U}\varphi = 2i\bar{h}\frac{d\varphi}{dt} + i\bar{h}\frac{dx}{dt}\frac{d\varphi}{dx} + i\bar{h}x\frac{d}{dt}\frac{d\varphi}{dx} \tag{122}$$

- We found an equation giving the relationship between wave-function, velocity, and displacement,

$$0 = \frac{i\bar{h}}{2m}\frac{d^2\varphi}{dx^2} + \frac{d\varphi}{dt} + v\frac{d\varphi}{dx} + x\frac{d}{dt}\frac{d\varphi}{dx} \tag{123}$$

- We found that the difference between phase velocity and group or classical velocity, according to our derived equation, is completely obeying our current knowledge of quantum mechanics.

$$\frac{\bar{h}k}{2m} = v_g - v_p \tag{124}$$

- We found a possible solution or explanation for wave-particle duality of quantum states.
- We derived relativistic equations of mass, length and time using the information given by our derived equation.

$$m = \frac{m_0}{\sqrt{(1 - \frac{v^2}{c^2})}} \tag{125}$$

$$l = l_0(1 - \frac{v^2}{c^2}) \tag{126}$$

$$\Delta t = \frac{\Delta t_0}{\sqrt{(1 - \frac{v^2}{c^2})}} \tag{127}$$

- We discovered the definition of the ability of a property to be a relativistic equation or we may say ***"Law of relativistic property".***

Conflict of Interest

Authors of this article declare that they have no conflict of interest.

Human Studies/Informed Consent

No human studies were carried out by the authors for this article.

Animal Studies

No animal studies were carried out by the authors for this article.

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